### The Elements of a Statistical Test

(1) Null hypothesis, $H_0$
(2) Alternate hypothesis, $H_a$
(3) Test statistic
(4) Rejection region (RR)

### Definition

A *type I error* is made if $H_0$ is rejected when $H_0$ is true. The *probability* of a *type I error* is denoted by $\alpha$. The value of $\alpha$ is called the *level* of the test.

A *type II error* is made if $H_0$ is accepted when $H_a$ is true. The *probability of a type II error* is denoted by $\beta$. .

### Examples

(A) Suppose that a political candidate, Jones, claims that he will gain more than 50% of the votes in a city election and thereby emerge as the winner. For Jones's political poll, $n = 15$ voters were sampled. We wish to test $H_0 : p = .5$ against the alternative, $H_a : p < .5$. The test statistic is $Y$ , the number of sampled voters favoring Jones. Calculate $\alpha$ if we select $RR = \{y \leq 2\}$ as the rejection region.

(B) Refer to Problem 1. Is our test equally good in protecting us from concluding that Jones is a winner if in fact he will lose? Suppose that he will receive 30% of the votes ($p = .3$). What is the probability $\beta$ that the sample will erroneously lead us to conclude that $H_0$ is true and that Jones is going to win?

(C) Refer to Problems 1 & 2. Calculate the value of $\beta$ if Jones will receive only 10% of the votes ($p = .1$).

### Large Sample $\alpha$-Level Hypothesis Tests

$$H_0 \,:\, \theta = \theta_0, \quad H_a = \begin{cases} \theta > \theta_0 & \text{upper-tail alternative} \\ \theta < \theta_0 & \text{lower-tail alternative} \\ \theta \neq \theta_0 & \text{two-tailed alternative} \end{cases} \quad \text{Test statistic} : Z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}} \quad \text{Rejection region} : \begin{cases} \{z > z_\alpha\} & \text{upper-tail RR} \\ \{z < -z_\alpha\} & \text{lower-tail RR} \\ \{|z| > z_{\alpha/2}\} & \text{two-tailed RR} \end{cases}$$

### Examples

(1) A vice president in charge of sales for a large corporation claims that salespeople are averaging no more than 15 sales contacts per week. (He would like to increase this figure.) As a check on his claim, $n = 36$ salespeople are selected at random, and the number of contacts made by each is recorded for a single randomly selected week. The mean and variance of the 36 measurements were 17 and 9, respectively. Does the evidence contradict the vice president's claim? Use a test with level $\alpha = .05$.

(2) A machine in a factory must be repaired if it produces more than 10% defectives among the large lot of items that it produces in a day. A random sample of 100 items from the day's production contains 15 defectives, and the supervisor says that the machine must be repaired. Does the sample evidence support his decision? Use a test with level .01.

(3) A psychological study was conducted to compare the reaction times of men and women to a stimulus. Independent random samples of 50 men and 50 women were employed in the experiment. The results are shown in the following table. Do the data present sufficient evidence to suggest a difference between true mean reaction times for men and women? Use $\alpha = .05$.

| Men | Women |
|---|---|
| $n_1 = 50$ | $n_2 = 50$ |
| $\overline{y}_1 = 3.6$ seconds | $\overline{y}_2 = 3.8$ seconds |
| $s_1^2 = .18$ | $s_2^2 = .14$ |

**The $p$-Value**

The *$p$-value* (or the *probability value*) associated with a test is the probability, under the null hypothesis, $H_0$, that the test statistic (a random variable) is equal to or exceeds the observed value (a constant) of the test statistic in the direction of the alternative hypothesis. Rather than select the critical region ahead of time, the $p$-value of a test can be reported and the reader then makes a decision.

(a) Refer to Problems (A) - (C) where $n = 15$ voters were sampled. If we wish to test $H_0 : p = .5$ versus $H_a : p < .5$, using $Y =$ the number of voters favoring Jones as our test statistic, what is the $p$-value if $Y = 3$? Interpret the result.

(b) Find the $p$-value for the statistical test of Problem (3).